

Backward hadron production in neutrino-nucleus interactions

O. Benhar^{1†}, S. Fantoni², G.I. Lykasov³

¹ *Jefferson Laboratory, Newport News, VA 23606, USA*

and

Department of Physics, Old Dominion University, Norfolk, VA 23529, USA

² *International School for Advanced Studies (SISSA)*

Via Beirut, 2-4. I-30014 Trieste, Italy

³ *Joint Institute for Nuclear Research, Dubna 141980, Moscow Region, Russia*

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Abstract

The production of backward pions in lepton-nucleus collisions is analyzed. We show that a large yield of high momentum backward pions can be explained by the Regge asymptotic of the distribution of nucleons carrying a large momentum fraction in the nuclear target. The calculated spectra of pions emitted in the $\nu + Ne \rightarrow \mu^- + \pi^\pm + X$ reaction are in satisfactory agreement with the available experimental data.

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Semi-inclusive lepton-induced hadron production reactions in which the detected hadron is emitted backward in the rest frame of the target have long been recognized as a powerful tool to investigate short-range nuclear dynamics. The studies recently carried out in refs. [1,2], focused on the emission of backward protons in e -A scattering, have shown that: i) the inclusion of high momentum components in the nucleon momentum distribution, $n(|\mathbf{k}|)$, is needed to account for the available data and ii) the behavior of $n(|\mathbf{k}|)$ at large $|\mathbf{k}|$ can be obtained from the Regge asymptotic of the proton spectra observed in soft hadron-nucleus collisions, provided the nonperturbative Q^2 -dependence of the quark distribution in the nucleus is taken into account [2].

Besides providing information on the structure of the nuclear wave function at short interparticle distance, the study of backward pion production in semi-inclusive lepton-nucleus processes allows to quantitatively investigate the fragmentation of the target nucleus into hadrons. In this paper we describe a theoretical calculation of the spectra of backward π^\pm arising from target fragmentation in neutrino-nucleus interactions, and compare the results to the experimental data of ref. [3].

The semi-inclusive spectrum of hadrons (e.g. pions) of energy E_h and three-momentum \mathbf{p} , detected in coincidence with a lepton of energy E' emitted in the direction specified by the solid angle Ω , can be written, within the framework of the impulse approximation, as

$$\begin{aligned} \rho_{\ell A \rightarrow \ell' h X}(q, \mathbf{p}) &= E_h \frac{d\sigma}{d\Omega dE' d\mathbf{p}} \\ &= \int d^4k S(k) r(k) \left[\frac{Z}{A} \rho_{\ell p \rightarrow \ell' h X}(k, q, \mathbf{p}) + \frac{N}{A} \rho_{\ell n \rightarrow \ell' h X}(k, q, \mathbf{p}) \right]. \end{aligned} \quad (1)$$

In the above equation q is the four-momentum transferred by the lepton, $S(k)$ is the relativistic-invariant function describing the nuclear vertex with an outgoing virtual nucleon of four-momentum k , $\rho_{\ell p \rightarrow \ell' h X}$ and $\rho_{\ell n \rightarrow \ell' h X}$ are the semi-inclusive spectra of hadrons h produced in ℓp and ℓn collisions, respectively, Z is the charge of the nucleus, A its atomic number and $N = A - Z$ the number of neutrons. The quantity $r(k)$ is the ratio of the fluxes associated with ℓN ($N = p, n$) and ℓA collisions.

The spectrum of eq.(1) can be rewritten in terms of the relativistic-invariant variable

$$z = \frac{M_A}{m} \frac{(pq)}{(P_A q)} , \quad (2)$$

where $p \equiv (E_h, \mathbf{p})$ is the four-momentum of the emitted hadron, m is the nucleon mass and M_A and P_A denote the target mass and four-momentum, respectively. The resulting expression is

$$\begin{aligned} \rho_{\ell A \rightarrow \ell' h X}(x, Q^2; z, p_t) = & \int_{z \leq y} dy \, d^2 k_t \, f_A(y, Q^2, k_t) \left[\frac{Z}{A} \rho_{\ell p \rightarrow \ell' h X} \left(\frac{x}{y}, Q^2; \frac{z}{y}, p_t - k_t \right) \right. \\ & \left. + \frac{N}{A} \rho_{\ell n \rightarrow \ell' h X} \left(\frac{x}{y}, Q^2; \frac{z}{y}, p_t - k_t \right) \right] . \end{aligned} \quad (3)$$

In the above equation, $Q^2 = -q^2$, x is the Bjorken scaling variable, $p_t = |\mathbf{p}|^2 - p_z^2$ and $k_t^2 = |\mathbf{k}|^2 - k_z^2$, p_z and k_z being the components of \mathbf{p} and \mathbf{k} along the direction of \mathbf{q} . The distribution function $f_A(y, Q^2, k_t)$ is defined as

$$f_A(y, Q^2, k_t) = \int dk_0 \, dk_z \, S(k) \, y \, \delta \left(y - \frac{M_A}{m} \frac{(kq)}{(P_A q)} \right) . \quad (4)$$

Note that the above definition includes the dependence of $f_A(y, Q^2, k_t)$ upon Q^2 , which was neglected in refs. [1,2] assuming the validity of the Bjorken limit. Hence, eq.(4) can be safely used at any value of Q^2 , including the region $Q^2 < 50 \text{ (GeV/c)}^2$, where the Q^2 -dependence of $f_A(y, Q^2, k_t)$ has been shown to be sizeable [4,5].

When the produced hadron is emitted backward, eq.(3) can be rewritten in the simplified form

$$\begin{aligned} \rho_{\ell A \rightarrow \ell' h X}(x, Q^2; z) = & \int_{z \leq y} dy \, f_A(y, Q^2) \left[\frac{Z}{A} \rho_{\ell p \rightarrow \ell' h X} \left(\frac{x}{y}, Q^2; \frac{z}{y} \right) \right. \\ & \left. + \frac{N}{A} \rho_{\ell n \rightarrow \ell' h X} \left(\frac{x}{y}, Q^2; \frac{z}{y} \right) \right] , \end{aligned} \quad (5)$$

with

$$f_A(y, Q^2) = \int d^4 k \, S(k) \, y \, \delta \left(y - \frac{M_A}{m} \frac{(kq)}{(P_A q)} \right) . \quad (6)$$

In principle, the distribution function $f_A(y, Q^2)$ can be calculated within nuclear many-body theory, approximating $S(k)$ with the nonrelativistic spectral function $P(k)$, yielding the probability of finding a nucleon with momentum \mathbf{k} and removal energy $(m - k_0)$ in the target

nucleus [6]. However, due to the limited range of momentum and removal energy covered by nonrelativistic calculations of $P(k)$ (typically $|\mathbf{k}| < .7 \text{ GeV}/c$ and $(m - k_0) < .6 \text{ GeV}$. See e.g. ref. [7]), this procedure can only be used in the region $y < 1.7$. An alternative approach to obtain $f_A(y, Q^2)$ at larger y , based on the calculation of the overlap of the relativistic-invariant phase-space available to quarks belonging to strongly correlated nucleon, has been recently proposed in ref. [1]. Within this approach the asymptotic behaviour of $f_A(y, Q^2)$ at $y > 1.7$ and small Q^2 can be related to the Regge asymptotic of the constituent quarks distribution at $y \rightarrow 1$ and the nonperturbative Q^2 -dependence is taken into account following ref. [8].

The second ingredient entering the calculation of the spectrum defined by eq.(3), i.e. the elementary semi-inclusive spectrum $\rho_{\ell N \rightarrow \ell' h X}(k, q, \mathbf{p})$, can be evaluated using the approach developed in refs. [9,10]. According to [9], the elementary production process can be described in terms of planar and cylindrical graphs in the s -channel, as shown in fig. 1 for the case of neutrino interactions, in which the exchanged particle is a W -boson. The planar graph of fig.1a describes the scattering of the incoming neutrino off a valence quark. Therefore, the corresponding contribution to the spectrum of fast backward hadrons, that will be denoted $F_P^N(x, Q^2; z)$, is proportional to the valence quark distribution multiplied by the fragmentation function of the spectator diquark into the detected hadron. In ref. [11] $F_P^N(x, Q^2; z)$ has been evaluated for the case of proton production in neutrino scattering, whereas the case of pion production in electron scattering has been discussed in ref. [1]. The planar graph contribution to the process $\nu + N \rightarrow \mu^- + \pi + X$ reads

$$F_P^N(x, Q^2; z) = z \phi_1(x, Q^2) \left[\frac{1}{3} D_{uu \rightarrow \pi} \left(\frac{z}{1-x} \right) + \frac{2}{3} D_{ud \rightarrow \pi} \left(\frac{z}{1-x} \right) \right] , \quad (7)$$

where

$$\phi_1(x, Q^2) = \frac{G^2 m E}{\pi} \frac{x}{1-x} \left(\frac{m_W^2}{m_W^2 + Q^2} \right) d_v(x, Q^2) . \quad (8)$$

In the above equation G is the Fermi coupling constant, E is the energy of the incoming neutrino, x is the Bjorken variable and d_v is the d -quark distribution. The functions $D_{uu \rightarrow \pi}$

and $D_{ud \rightarrow \pi}$ describe the fragmentation of the spectator diquark, uu or ud , into a positive or negative pion, whereas m_W is the W -boson mass.

It is well known that, in addition to the dependence upon x and $z/(1-x)$, the quark distributions and fragmentation functions display a Q^2 dependence. For small Q^2 they exhibit true Regge asymptotic [9,10] at $x_F \rightarrow \pm 1$, $x_F = 2p_z/W_X$ and W_X being the Feynman variable and the invariant mass of the undetected debris, respectively. This Regge asymptotic, discussed in the Appendix, is mainly determined by the intercepts of the Reggeon ($\alpha_R(0)$) and the averaged baryon trajectories ($\tilde{\alpha}_B(0)$) and is completely different from the corresponding behaviour observed in deep inelastic lepton-nucleon scattering, where Q^2 is large [9,10]. According to refs. [9] and [10] the planar graph of fig.1a corresponds to the one-Reggeon graph in the t -channel, having the asymptotic behaviour $W_X^{\alpha_R(0)-1}$, where $\alpha_R(0) = 1/2$ is the Reggeon intercept. As a consequence, the contribution of this graph is a decreasing function of W_X .

The contribution of the so called cylindrical graph, associated with scattering off a sea quark and shown in fig.1b, will be denoted $F_C(x, Q^2; z)$. It can be written in the form [1,10,12]

$$F_C(x, Q^2; z) = z \phi_2(Q^2) [L_1(z, x) + L_2(z, x)] , \quad (9)$$

where

$$\phi_2(Q^2) = \frac{G^2 m E}{\pi} \left(\frac{m_W^2}{m_W^2 + Q^2} \right) , \quad (10)$$

$$L_1 = \int_z^{1-x} \left[u_v(y) D_{u_v \rightarrow \pi} \left(\frac{z}{y} \right) + d_v(y) D_{d_v \rightarrow \pi} \left(\frac{z}{y} \right) \right] , \frac{dy}{y} \quad (11)$$

and

$$L_2 = \int_z^{1-x} \left\{ \frac{4}{3} f_{ud}(y) D_{ud \rightarrow \pi} \left(\frac{z}{y} \right) + \frac{1}{3} \left[f_{uu} D_{uu \rightarrow \pi} \left(\frac{z}{y} \right) + f_{dd}(y) D_{dd \rightarrow \pi} \left(\frac{z}{y} \right) \right] \right\} \frac{dy}{y} . \quad (12)$$

In the above equations u_v is the distribution of the valence u -quark, whereas f_{uu} , f_{ud} and f_{dd} are the distributions of uu -, ud - and dd -diquarks. $D_{u_v \rightarrow \pi}$, $D_{d_v \rightarrow \pi}$ and $D_{dd \rightarrow \pi}$ are the fragmentation functions of the valence u and d quarks and the dd diquark into pions, respectively. The expressions of the quark and diquark distributions and fragmentation functions

employed in our calculations, obtained within the approach of ref. [10], are given in the Appendix. At small Q^2 the cylindrical graph of fig.1b corresponds to one-Pomeron exchange in the t -channel, having the asymptotic behaviour $W_X^{\tilde{\alpha}_P(0)-1}$. For the supercritical Pomeron the value of $\tilde{\alpha}_P(0)$ is given by the relation $\Delta \equiv \tilde{\alpha}_P(0) - 1 \simeq 0.08$ [9,10]. Comparison between the W_X -dependence of the cylindrical and planar graphs of fig.1 shows that the contribution of the planar graph can be neglected at large W_X and not too large Q^2 .

The functions $\phi_1(x, Q^2)$ and $\phi_2(x, Q^2)$ describe the upper vertices of figs. 1a and 1b. In the case of neutrino-nucleon interaction they are proportional to the charged electroweak current. In conclusion, the relativistic-invariant spectrum $zd^3\sigma/dxdzdp_t$ of pions produced in $\nu + N \rightarrow \mu^- + \pi^\pm + X$ processes can be written in the form

$$z \frac{d^3\sigma}{dxdzdp_t} = F_P(x, Q^2; z, p_t) \left(\frac{W_X}{s_0} \right)^{\alpha_R(0)-1} + F_C(x, Q^2; z, p_t) \left(\frac{W_X}{s_0} \right)^{\tilde{\alpha}_P(0)-1}, \quad (13)$$

where $s_0 = 1. \text{GeV}^2$ is a parameter usually introduced in Regge theory in order to get the correct dimension of the matrix elements or cross sections. Eq.(13) clearly shows that the cylindrical graph (fig. 1b) provides the main contribution to the spectrum at large W_X .

The elementary spectrum $\rho_{\nu N \rightarrow \mu^- \pi^\pm X}$ corresponding to backward production can be readily obtained from the above $zd^3\sigma/dxdzdp_t$. Substituting into eq.(5) one can finally evaluate the semi-inclusive spectrum for the case of neutrino-nucleus scattering and compare to the available ν -Ne data, taken at CERN by the WA 59 collaboration [3].

In order to compare to the data, the calculated spectrum has to be integrated over x and Q^2 . Since the main contribution to the integral comes from the region of small Q^2 and large ν/s (ν denotes the neutrino energy loss, while $s = (q + k)^2$), implying in turn large W_X , we have included in the calculation only the contribution of the cylindrical graph of fig. 1b. The calculated $(E/\sigma)\rho_{\nu+N \rightarrow \mu^-+\pi^\pm+X}(p_\pi^2)$, where p_π and σ are the magnitude of the pion momentum and the total cross section, respectively, is shown in fig. 2 together with the experimental data of ref. [3]. For comparison, we also show (dashed line) the results of a calculation carried out using the spectral function of ref. [7], which vanishes for $k > 0.8 \text{ GeV}/c$, to evaluate the distribution function of eq.(6). The large difference between the

dashed and solid lines of fig. 2, particularly at high pion momenta, $p_\pi > 0.4$ GeV/c, indicates that the contribution of the kinematical region corresponding to nucleon momenta larger than ~ 0.8 GeV/c dominates. The shape of the pion spectra at large momenta turns out to be determined by the asymptotic behaviour of the nucleon distribution f_A as a function of the relativistic invariant variable $y = (M_A/m)(kq)/(P_Aq)$. As a consequence, it appears that semi-inclusive lepton-nucleus and hadron-nucleus reactions in the kinematical region forbidden to free ℓN or hN scattering can be related to each other.

In this paper we have applied the approach developed in ref. [1] to construct the distribution function $f_A(y, Q^2)$ (see eq.(6)) in the region of very large y ($y > 1.7$), dominated by short range nuclear dynamics. Short range nucleon-nucleon correlations are described in terms of overlapping distributions of three-quark colorless objects. The conventional treatment of nucleon-nucleon correlations, based on the spectral function obtained within nonrelativistic many-body theory, cannot be employed to obtain $f_A(y, Q^2)$ at $y > 1.7$, the nucleon momenta involved being too large ($> .8$ GeV/c). Our approach, originally proposed to describe hadron production in hadron-nucleus processes refs. [13–15], quantitatively accounts for the spectra of backward pions produced in the $\nu + Ne \rightarrow \mu^- + \pi^\pm + X$ process. Thus, our results suggest that the same mechanism is responsible for both hadron- and lepton-induced emission of fast backward hadrons from nuclear targets.

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APPENDIX A:

The distributions of valence quarks ($q_v(z)$) and diquarks ($f_{qq}(z)$) (q denotes either the u or d quark), exhibiting true Regge asymptotic at small Q^2 , can be written according to [10]:

$$q_v(z) = C_q z^{-\alpha_R(0)} (1-z)^{\alpha_R(0)-2\tilde{\alpha}_B(0)} , \quad (\text{A1})$$

and

$$f_{qq}(z) = C_{qq} z^{\alpha_R(0)-2\tilde{\alpha}_B(0)} (1-z)^{-\alpha_R(0)} . \quad (\text{A2})$$

The coefficients C_q and C_{qq} are determined by the normalization conditions

$$\int_0^1 q_v(z) dz = \int_0^1 f_{qq}(z) dz = 1 . \quad (\text{A3})$$

The fragmentation functions have the form [10]

$$D_{u_v \rightarrow \pi^+} = \frac{a_0}{z} (1-z)^{\alpha_R(0)+\lambda} , \quad (\text{A4})$$

$$D_{u_v \rightarrow \pi^-}(z) = (1-z) D_{u_v \rightarrow \pi^+}(z) , \quad (\text{A5})$$

$$D_{d_v \rightarrow \pi^+}(z) = D_{u_v \rightarrow \pi^-}(z) , \quad (\text{A6})$$

$$D_{d_v \rightarrow \pi^-}(z) = D_{u_v \rightarrow \pi^+}(z) , \quad (\text{A7})$$

$$D_{uu \rightarrow \pi^+} = \frac{a_0}{z} (1-z)^{\alpha_R(0)-2\tilde{\alpha}_B(0)+\lambda} , \quad (\text{A8})$$

$$D_{uu \rightarrow \pi^-}(z) = (1-z) D_{uu \rightarrow \pi^+}(z) , \quad (\text{A9})$$

$$D_{ud \rightarrow \pi^+}(z) = D_{ud \rightarrow \pi^-}(z) = \frac{a_0}{z} [1 + (1-z)^2] (1-z)^{\alpha_R(0)-2\tilde{\alpha}_B(0)+\lambda} , \quad (\text{A10})$$

$$D_{dd \rightarrow \pi^-}(z) = D_{uu \rightarrow \pi^+}(z) , \quad (\text{A11})$$

and

$$D_{dd \rightarrow \pi^+}(z) = D_{uu \rightarrow \pi^-}(z) i . \quad (\text{A12})$$

In the above equations $\alpha_R(0) = 0.5$ is the intercept of the Reggeon trajectory, $\tilde{\alpha}_B(0) = -0.5$ is the intercept of the average baryonic trajectory, $a_0 = 0.65$ and $\lambda = 2\alpha'_R(0) < p_t^2 > \simeq 0.5$, $\alpha'_R(0)$ and $< p_t^2 >$ being the slope of the Reggeon trajectory and the average value of the transverse hadron momentum squared.

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[†] Present address: INFN, Sezione Roma 1, I-00185 Rome, Italy

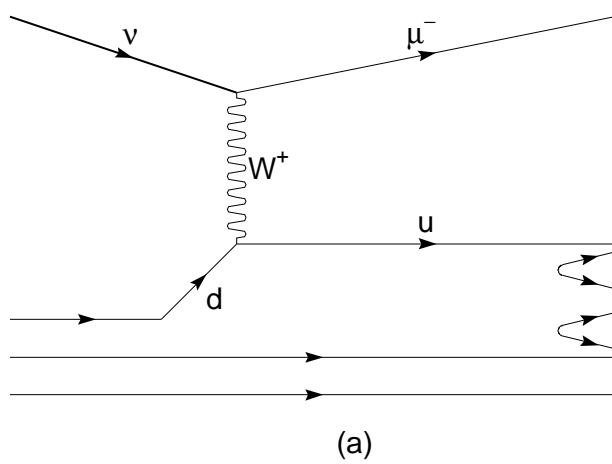
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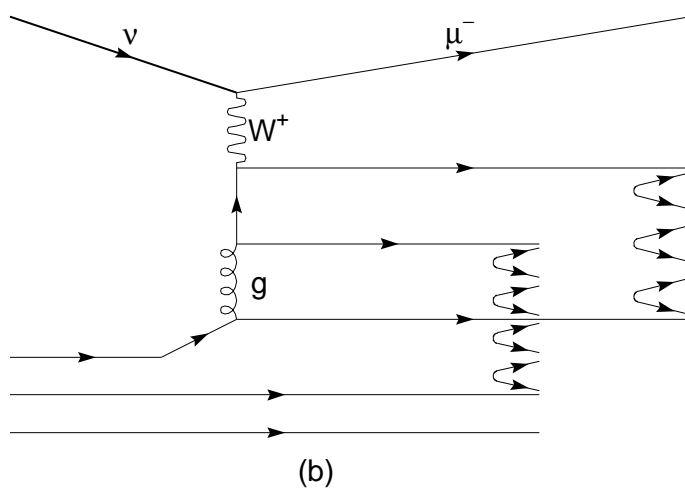
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FIGURES

FIG. 1. Planar (a) and cylindrical (b) graphs contributing to the reaction $\nu + N \rightarrow \mu^- + h + X$. Diagrams (a) and (b) describe processes in which the incoming neutrino interacts with a valence quark or a sea quark (or antiquark), respectively.

FIG. 2. (a): p_π^2 -dependence of the spectrum of π^- -mesons produced backward in the $\nu + Ne \rightarrow \mu^- + \pi^- X$ process. The solid line has been obtained using the approach described in this paper, whereas the dashed curve shows the results of a calculation carried out using the nuclear spectral function $P(k)$ of ref. [7]. The experimental data are taken from ref. [3]. (b): same as in (a) but for π^+ .





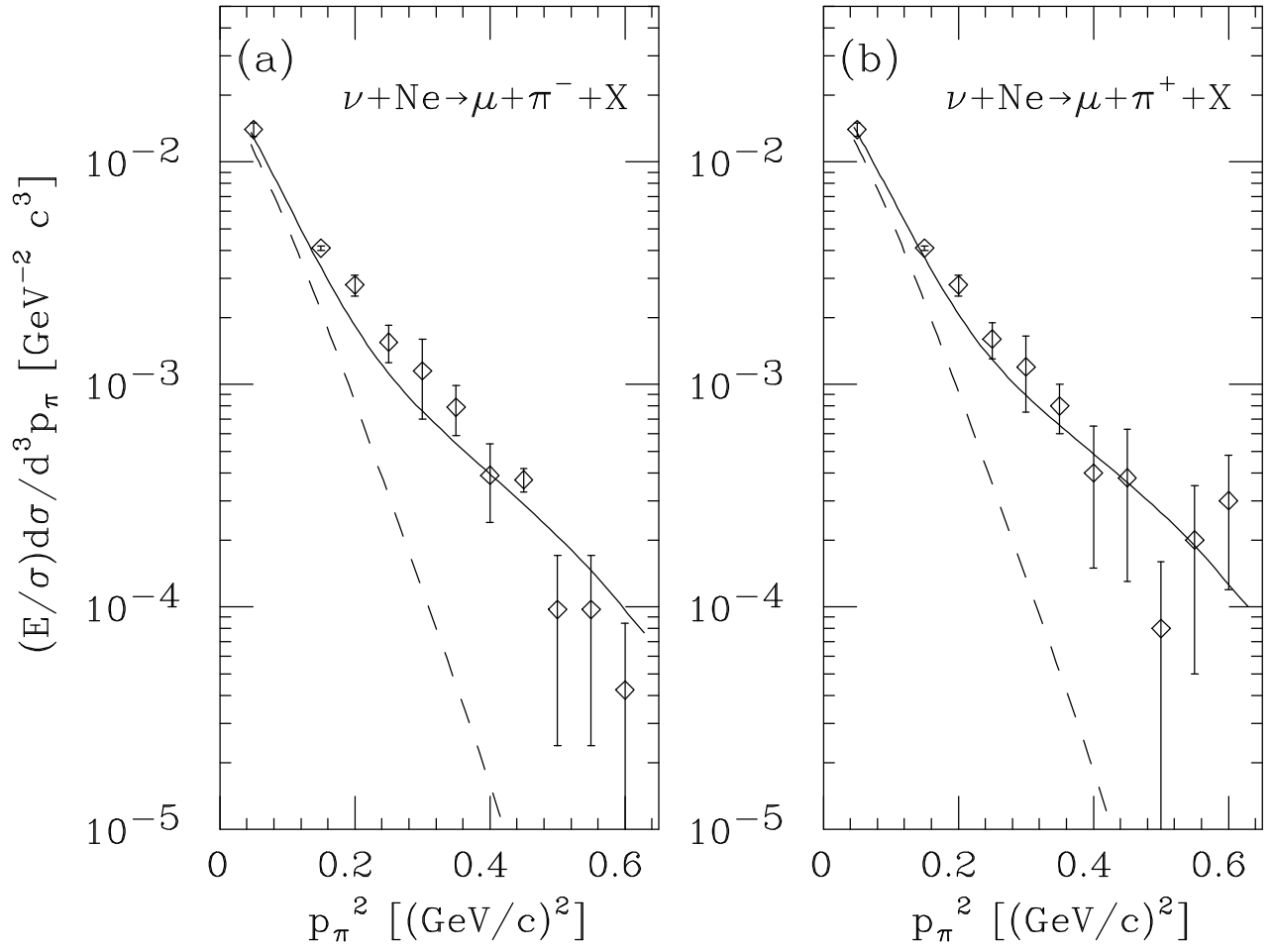


Fig. 2